

**Grade Level/Course:** Grades 3 and 4

**Lesson/Unit Plan Name:** Recognizing and Generating Equivalent Fractions

**Rationale/Lesson Abstract:** Students draw models and number lines to represent equivalent fractions.

**Timeframe:** 9 Days for Grade 3 : Part 1: 2 Days , Part 2: 2 days, Part 3: 5 Days

**Common Core Standard(s):**

CCSS.Math.Content.3.NF.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

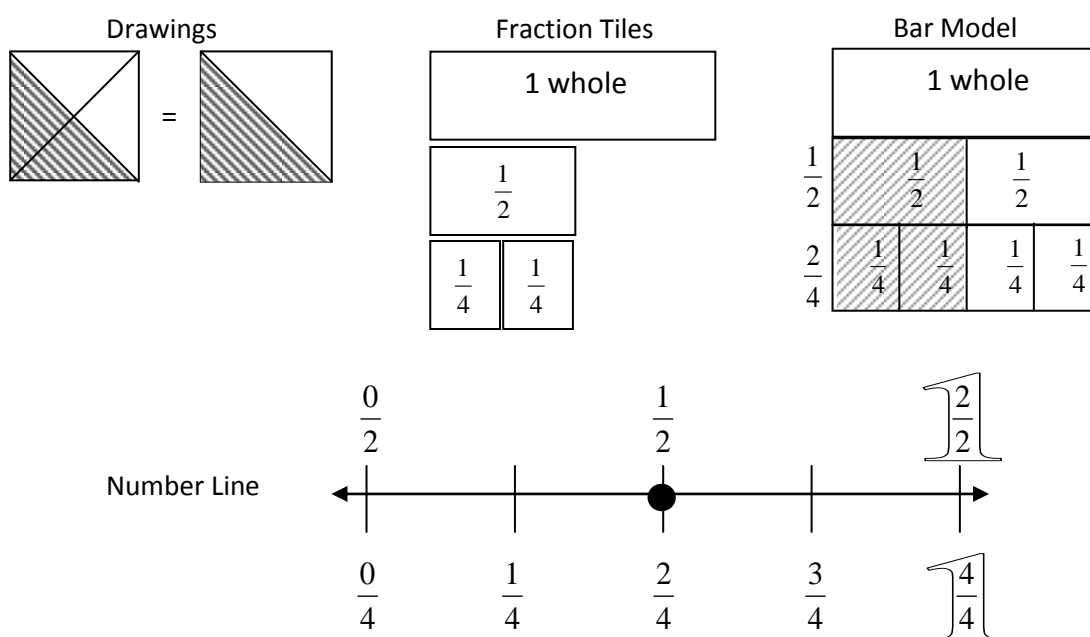
a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g.,  $\frac{1}{2} = \frac{2}{4}$ ,  $\frac{4}{6} = \frac{2}{3}$ . Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. (<sup>1</sup> Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, 8.)

CCSS.Math.Content.4.NF.1: Explain why a fraction  $\frac{a}{b}$  is equivalent to a fraction  $\frac{n \times a}{n \times b}$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions (<sup>1</sup> Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 100.)

These lessons assume that students have drawn bar models and plotted fractions on number lines. Models Used in these Lessons:



**Instructional Resources/Materials:** Paper, pencil, straight edge, fraction tiles

### Justification of Methods Used:

When students see and work with equivalent fractions using different models, they develop a deeper understanding of the concept.

This falls in line with Common Core.

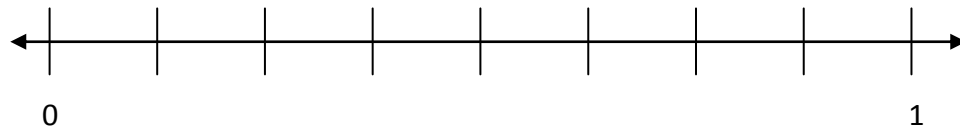
Example: Grade 3 SBAC Practice Test (2013)

Drag each fraction to the correct location on the number line....

$$\frac{1}{2}$$

$$\frac{1}{4}$$

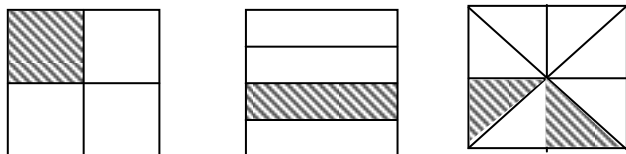
$$\frac{1}{8}$$



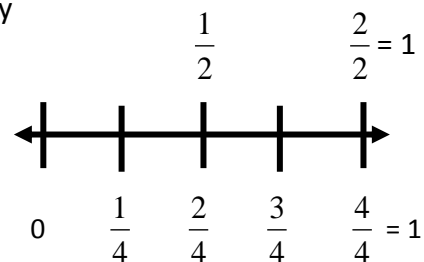
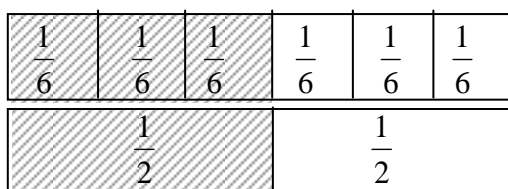
Example: Some images reproduced from **Progressions for the Common Core State Standards in Mathematics (draft)** by The Common Core Standards Writing Team Sept, 2013

(Grade 3)

Some Area Representations of  $\frac{1}{4}$ : (There are more in the Progressions documents.)



Number Line and Fraction Strips to Show Equivalency



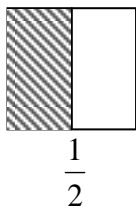
Activity/Lesson:

### Part 1: Equivalent Fractions Using Model Drawings (2 days)

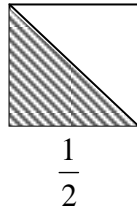
Student notes may look something like this after the first lesson, which is described on the next page. Some students will draw only a few examples, and some will draw many. Ideally, students would write and draw everything in their notebooks, being as accurate as they can (perfection not expected). The more they draw and create, the better. However, there are square templates at the back of the lesson if the teacher wants to use them. Also, use of the word, “partitioned” is used, keeping consistent with Common Core language.

#### Equivalent Fractions:

Sasha's Pizza:

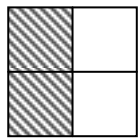


or

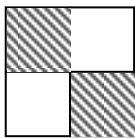


Cooper's Pizza:

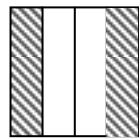
$$\frac{2}{4} = \frac{1}{2}$$



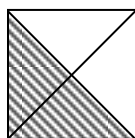
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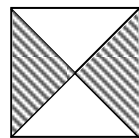
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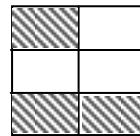
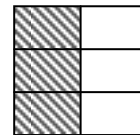
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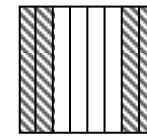
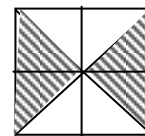
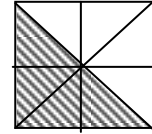
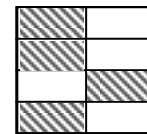
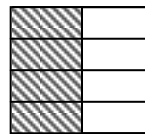
or



$$\frac{3}{6} = \frac{1}{2}$$



$$\frac{4}{8} = \frac{1}{2}$$



**Equivalent fractions** are the same value (or amount), but they look different.

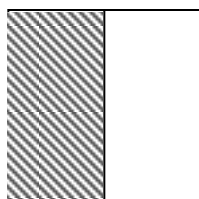
$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}, \text{ so they are equivalent.}$$

**Introduction Story:** Sasha and Cooper each had a mini pizza. Sasha's was cut, or partitioned, into halves and Cooper's was partitioned into fourths. Sasha only ate one piece, but Cooper ate 2 pieces. Who ate more pizza? Let's find out. (or) Let's prove it.

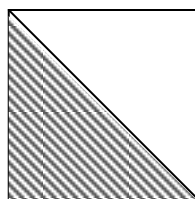
On your paper draw 2 squares about the same size. They don't have to be perfect, but you should try to make them the same if you can. Each square represents 1 whole pizza.

If Sasha had a whole pizza that was partitioned into halves, and she ate one of the pieces, it could look something like this, right? Draw and shade one half in each of the pizzas.

Sasha's pizza:



OR

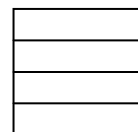
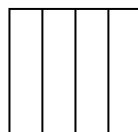
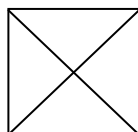


$$\frac{1}{2}$$

$$\frac{1}{2}$$

Now draw a horizontal line under your work. We're going to draw Cooper's pizza.

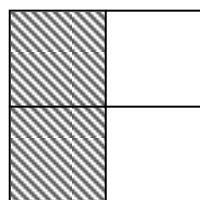
(Discuss and draw different ways to make fourths on the board. Choose which one to start with. I chose the first one.)



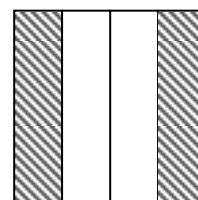
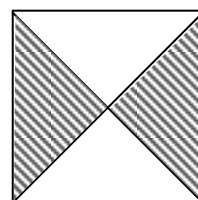
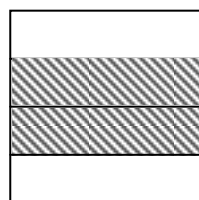
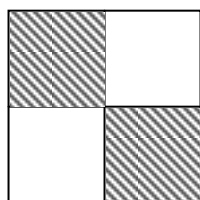
On your page, draw 2 square pizzas about the same size, and divide them into fourths.

How many pieces did Cooper eat? (2) We can say 2 pieces or 2 fourths. Let's shade in 2 fourths of each new pizza.

Cooper's Pizza:



or



Does this look a lot like the half that Sasha ate? (Yes,) So they ate the same amount. So  $\frac{2}{4} = \frac{1}{2}$ .

Does it matter which 2 pieces Cooper ate? (no) It just needs to be 2 of the fourths. It doesn't matter which 2.

Go ahead and draw a different way to show 2 fourths. Thumbs up when you're done. (Students can use the same fourths model or a new one.)

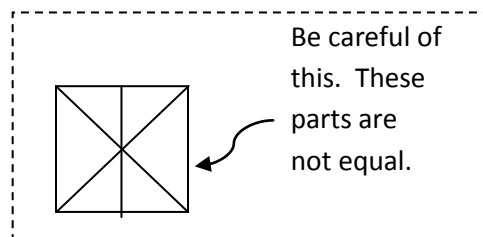
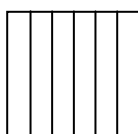
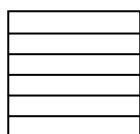
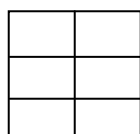
Okay, now draw as many different ways to make 2 fourths as you can. Remember to start with a square about the same size each time. (Share out drawings.)

(Label this section  $\frac{2}{4} = \frac{1}{2}$ .)

Draw a line under your work or go to the next page to work with something new. What if we cut, or partitioned, the pizzas into sixths?

**We Do:** How many sixths would be the same as 1 half? (3) Why 3? (3 is half of 6)

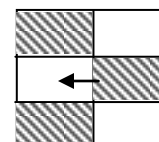
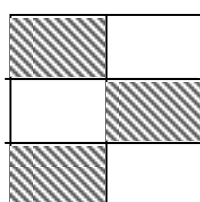
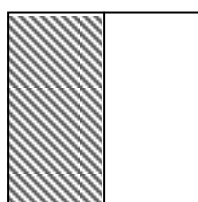
What are different ways to make sixths? (on the board...)



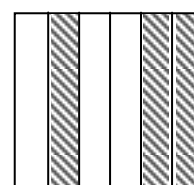
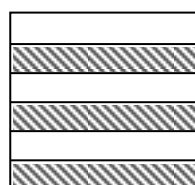
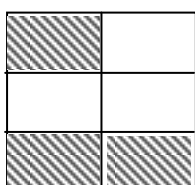
(Choose a model and shade in 3). Does this look like 1 half?

What about these 3? (Yes)

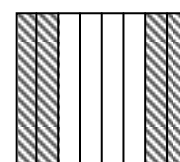
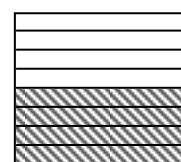
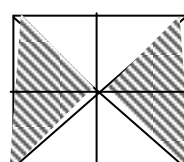
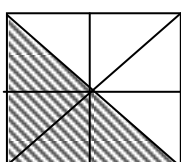
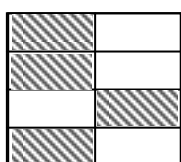
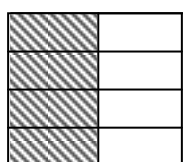
Another approach: Can you imagine the middle piece moved over?



Thumbs up if you can come up with your own ways of shading 3 sixths. Have students make and share drawings. Possible student drawings:



Draw a line under your work. Now we will work with eighths. (Follow a similar procedure with eighths.) Possible Student drawings:



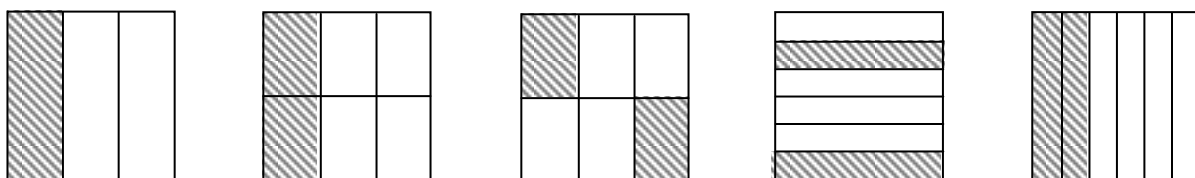
Since  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$  we say that these fractions are *equivalent*.

**Equivalent fractions** are the same value (or amount), but they look different. We will be working with equivalent fractions for a while. Sometimes people say equal instead of equivalent.

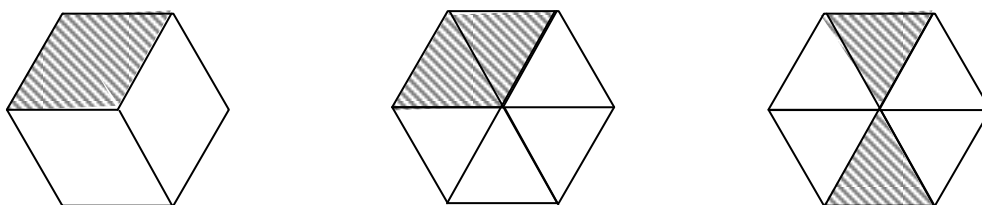
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In the next session, students draw equivalent fractions for 1 third, 1 fourth, 2 thirds, and 3 fourths. Focusing mainly on the unit fractions 1 third and 1 fourth will provide foundation for the more complex fractions.

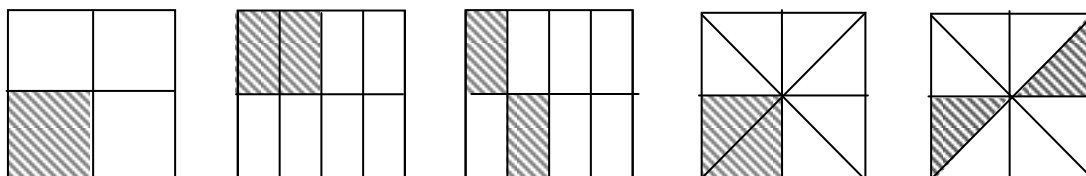
$$\frac{1}{3} = \frac{2}{6} \quad \text{or} \quad 1 \text{ third} = 2 \text{ sixths} \quad \text{or} \quad \text{There are 2 sixths in 1 third.}$$



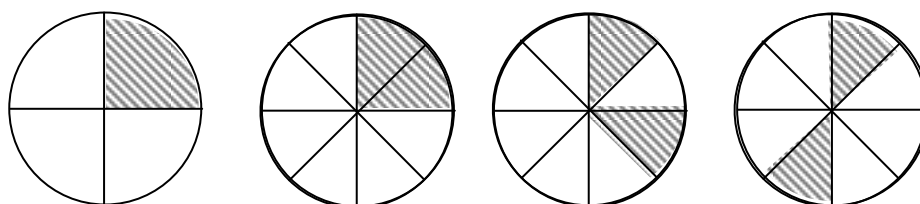
The shape of the whole is not restricted to squares.



$$\frac{1}{4} = \frac{2}{8} \quad \text{or} \quad 1 \text{ fourth} = 2 \text{ eighths} \quad \text{or} \quad \text{There are 2 eighths in 1 fourth.}$$



Another model:

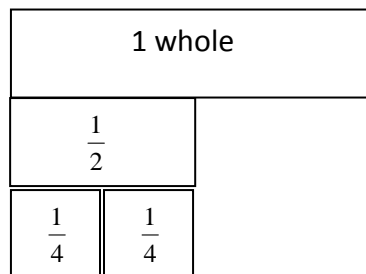


## Part 2: Generating Equivalent Fractions using Fraction Tiles and Bar Models (2 Days)

### Fraction Tiles (Day 1)

Have students use the fractions pieces in their fraction tile set to see which can be used to make 1 half.

*One Approach:* Have students set up 1 whole with 1 half below it. Then place the different fraction tiles below the 1 half to see if they line up.



Record the class findings: How many thirds? (Thirds did not work.) Why? (1 third was not enough, and 2 thirds was too much.) What about fourths? (2 fourths) Show us. Let's put that on the list. What else worked? (Continue until all fractions equivalent to 1 half are listed.)

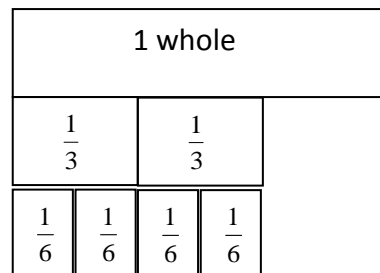
You can do the same with other unit fractions (1 third, 1 fourth). You can also have them find equivalents for 2 thirds and 3 fourths.

Example: After students have explored, ask them....

What worked? (sixths) How many sixths? (4) Show us.

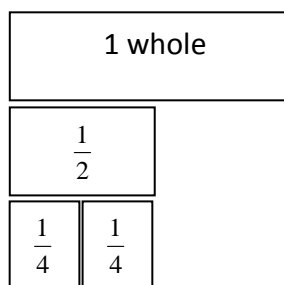
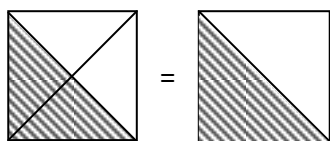
Did anything else work? (If your set has ninths and twelfths, you can talk about those.)

Why didn't halves work? Fourths?



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### Connect Fraction Tiles and Drawings:

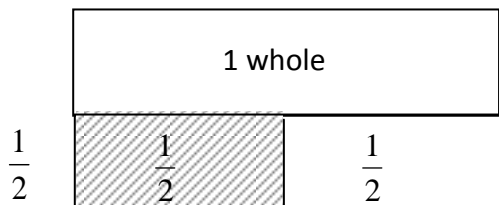


*(Display the drawing and the fraction tiles side by side. The purpose is to develop a deeper understanding of equivalency by comparing different models.)* What is the same about the drawing and the fraction tiles? (Both show 1 half; both have fourths). Prompt students as necessary: Where is the whole in the drawing? In the fraction tiles? Where is 1 half? Where are the 2 fourths?

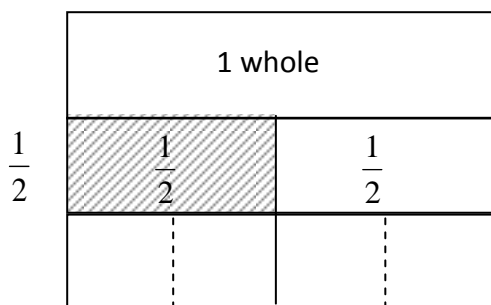
What is different? (Fraction tiles are things you can move.) Which is easier to work with?

## Bar Models (Day 2)

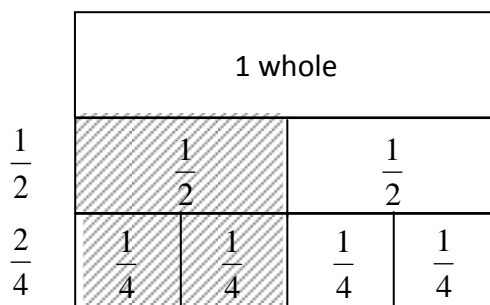
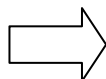
Today we're going to draw bar models and number lines to match what we did with the fraction tiles. (Students can have fraction tiles with them, or you can have a list that was made the previous day showing what students discovered using the fraction tiles.)



Do we all agree that 1 half equals 2 fourths? Let's draw it. Start with one whole. Then make (partition into) 2 halves and shade in 1 half.

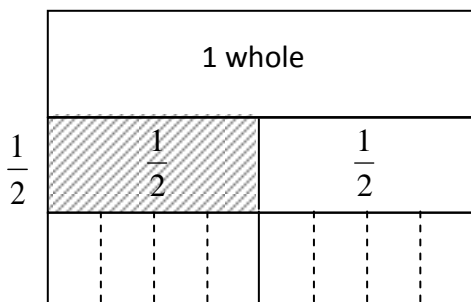


If we extend each half down to the next layer, we will have 2 parts, but we need 4 parts. What should we do? If we divide each half into 2 pieces, then will we have 4? (Yes!)

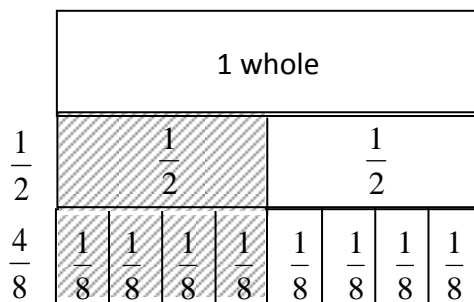


Now we can label each 1 fourth. If we shade 2 of them, we can show how they line up with 1 half.

**We Do:** How many eighths in 1 half?  $\frac{1}{2} = \frac{?}{8}$  Our list says 4 eighths. Let's draw it.



Start with 1 whole and 1 half, just like before. Extend the halves down. Divide, or partition, each half into fourths, and then eighths.



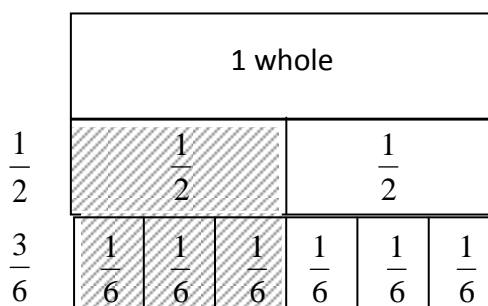
Label and shade in eighths until you get to 1 half.

$$\therefore \frac{1}{2} = \frac{4}{8}$$



**You Try:** How many sixths are in 1 half?

$$\frac{1}{2} = \frac{?}{6}$$



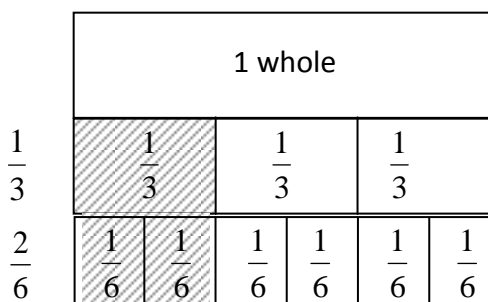
$$\therefore \frac{1}{2} = \frac{3}{6}$$

**You Try:** What about 1 third?

How many sixths are in 1 third?

$$\frac{1}{3} = \frac{?}{6}$$

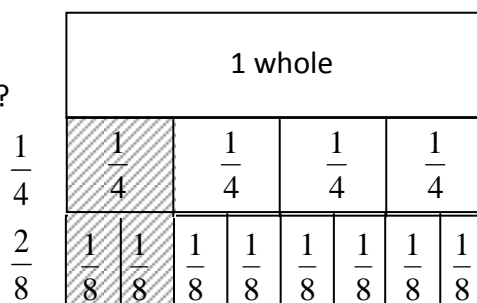
(When students are confident with the bar model, you can take the list down, so they have to rediscover the equivalents on their own. The bar model becomes a tool at that point.)



$$\therefore \frac{1}{3} = \frac{2}{6}$$

**You Try:** How many eighths are in 1 fourth?

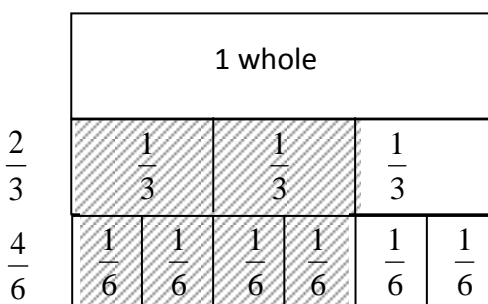
$$\frac{1}{4} = \frac{?}{8}$$



$$\therefore \frac{1}{4} = \frac{2}{8}$$

**You Try:** How many sixths are in 2 thirds?

$$\frac{2}{3} = \frac{?}{6}$$



$$\therefore \frac{2}{3} = \frac{4}{6}$$

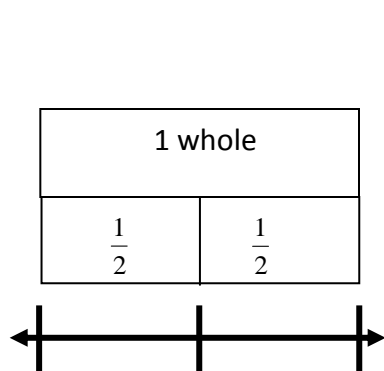
**Other Possible We Do/You Tries:** How many sixths are in 3 thirds?(6)\* How many fourths are in 2 halves?(4) How many eighths are in 2 fourths?(4) How many eighths are in 3 fourths?(6) How many eighths are in 4 fourths?(8) How many tenths in 1 fifth? (2)

\*If you have the students work with “How many sixths are in 3 thirds?” you can revisit equivalent forms of one. “We know 3 thirds = 1 whole. How many sixths make 1 whole? (6). So shouldn’t they be equal to each other? Let’s find out.”

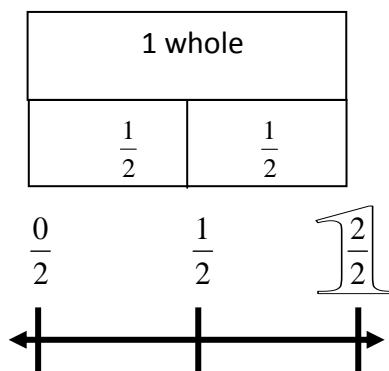
### Part 3: Bar Models and Number Lines (4 days)

#### Connect Fraction Bars (previous model) to Number Line (new model) (Days 1 and 2):

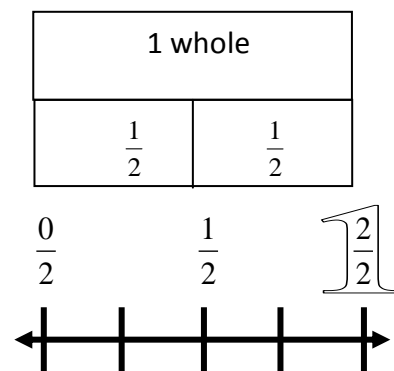
This demonstration on the board can help students understand the double number line.



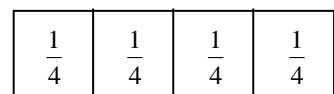
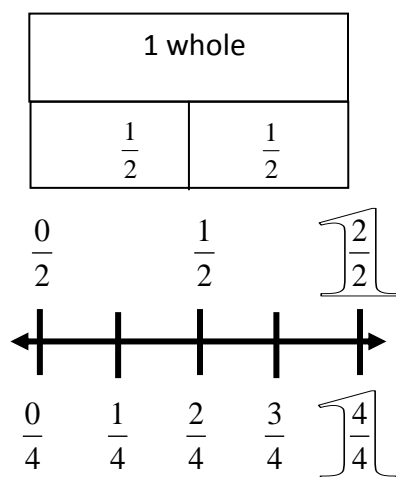
Tape large fraction strips of 1 whole to the board. Draw a number line below it and mark off 1 whole. Then move the 1 whole up and add the halves. Mark off 1 half on the number line.



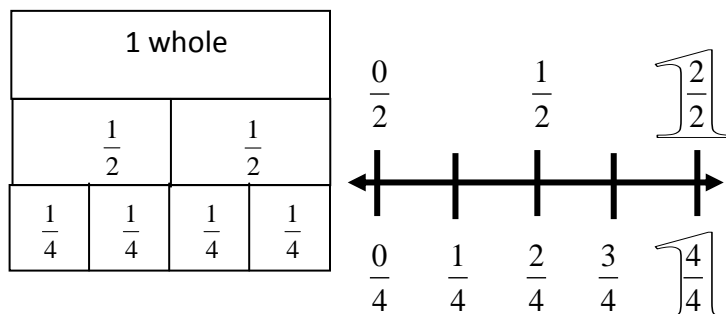
Move the strips up and label the halves above the number line.



Add the fourths strips below the number line and mark off fourths.



Move the fourths fraction strip down and label the fourths below the number line.



Move the fractions strips to the side, aligning the number line between the halves and fourths, so students can see both models side by side. Compare: What's the same about the number line and fraction strips? What's different? Where are the halves in each? Where's the whole? Where's the fourths?

Have students use their fraction strips or fraction tiles to create number lines in the same manner.

We Do: halves and fourths

You Try: halves and eighths

You Try: sixths and thirds

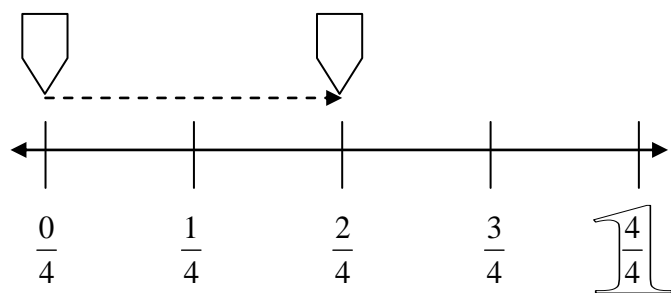
You Try: halves and sixths

You Try: fourths and eighths

\*It doesn't matter which unit is on the top. Students may do a few you tries or many. Accuracy will vary. The important thing is that they develop an understanding of the number line.

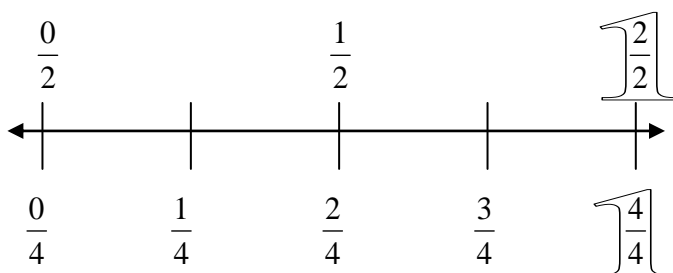
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**Side Note: Another Way to Look at It:**



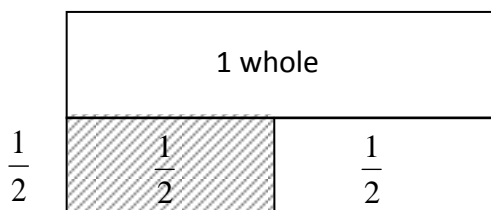
Establish one whole divided into fourths.

Run your pencil along the number line.  
Have the students raise their hands when  
you get half way across.

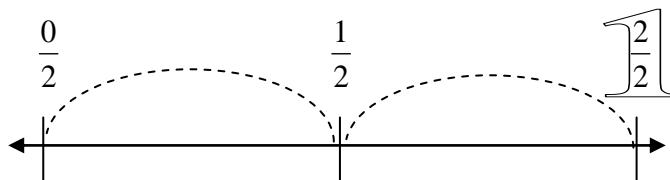


Then label halves on the top of the line .

**Bar Model and Number Line (Days 3 and 4 ):** This requires students to partition units without the physical guide of a fraction tile. Students will benefit from trying to partition equal units, even if accuracy is not quite there. Teacher notes should have the bar model side by side with the number line. It's ideal for students to do the same.

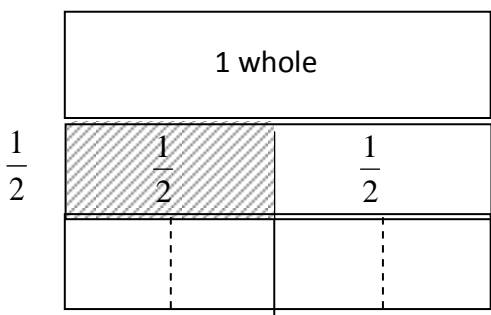


First we make 1 whole. Then we extend the whole down and partition it into halves. Then we shade in 1 half.

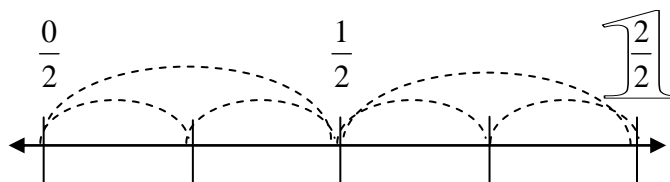


We also partition the number line into halves and label each half in order ( $\frac{0}{2}, \frac{1}{2}, \frac{2}{2}$ ).

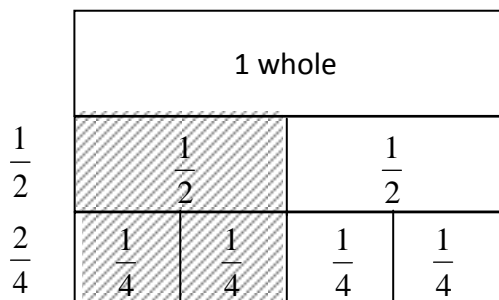
Thumbs up if you're done. Remember to check your neighbor's work.



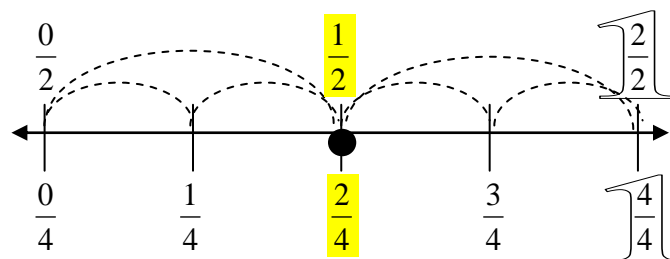
Then we extend the halves down and partition them into fourths.



On a number line, we can partition the halves into fourths also. We put 2 jumps in each half to make fourths. Thumbs up when you have that done.



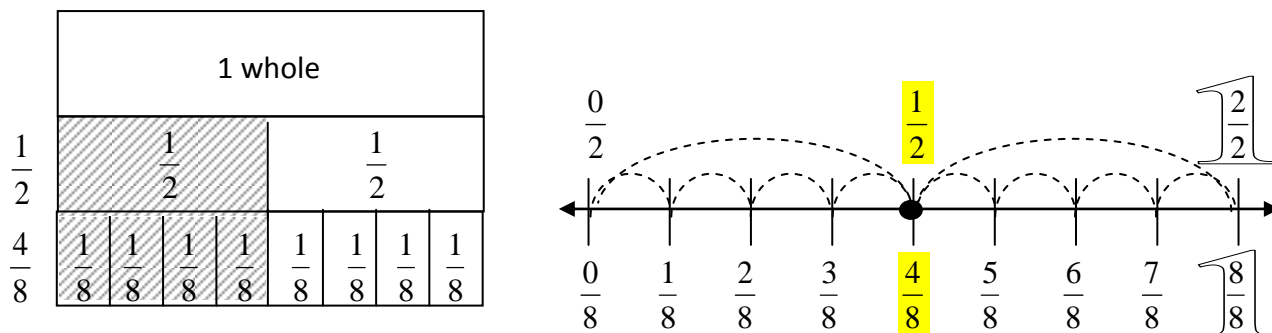
Finally, we can label all the fourths and shade in 2 fourths.



On the number line, we can label the fourths below the line. Then we can plot 1 half and 2 fourths all at once because they are in the same spot.

**We Do #2:** How many eighths are in 1 half?  $\frac{1}{2} = \frac{?}{8}$

Give less guidance on this go around.



$\therefore$  There are 4 eighths in 1 half, or  $\frac{1}{2} = \frac{4}{8}$  (Either way of answering is fine.)

**You Try:** How many sixths are in 1 half?  $\frac{1}{2} = \frac{?}{6}$ . Draw a bar model and number line.

**Other Possible You Tries (similar to those from before)** If you think the students are ready, they can just create the number line without a bar model.

How many sixths are in 1 third?  $\frac{1}{3} = \frac{?}{6}$

How many eighths are in 3 fourths?  $\frac{3}{4} = \frac{?}{8}$

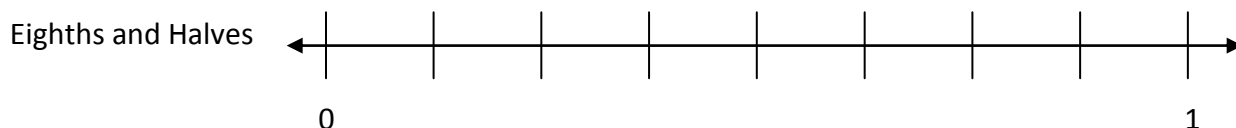
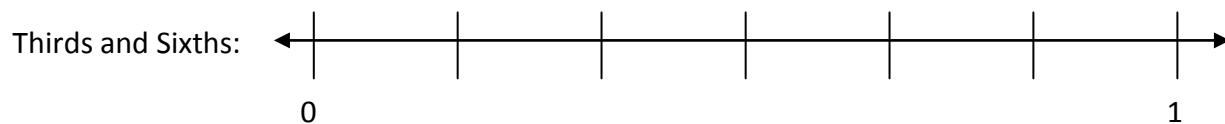
How many eighths are in 1 fourth?  $\frac{1}{4} = \frac{?}{8}$

How many sixths are in 3 thirds?  $\frac{3}{3} = \frac{?}{6}$

How many sixths are in 2 thirds?  $\frac{2}{3} = \frac{?}{6}$

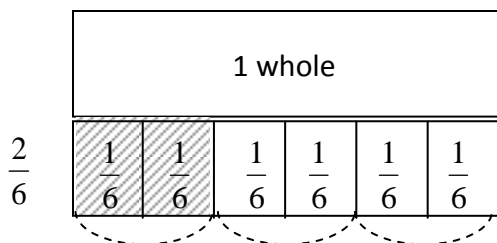
How many eighths are in 4 fourths?  $\frac{4}{4} = \frac{?}{8}$

Possible follow up warm up questions and exercises: Student label each number line as indicated.

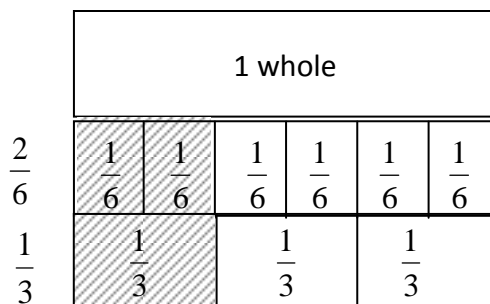


## Bar Model and Number Line: From Smaller Units to Greater Units (Day 5)

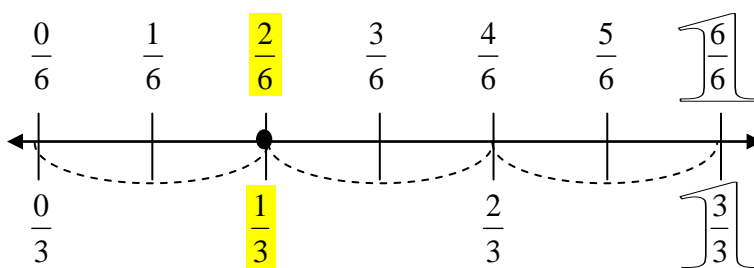
**We do:** How many thirds are in 2 sixths?  $\frac{2}{6} = \frac{?}{3}$



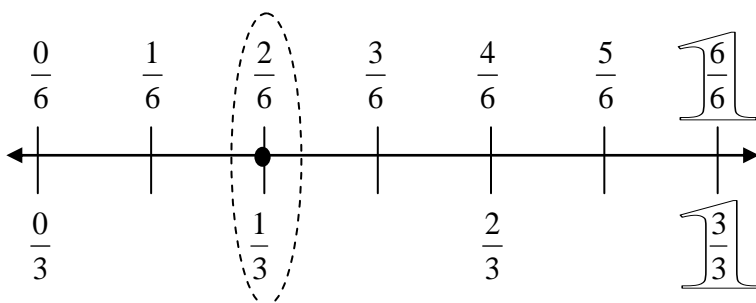
I have 6 equal parts, or sixths. I need to make only 3 equal parts, or thirds. Can we put these 6 pieces into 3 groups?



Then label each third. How many thirds will we shade in? (1)

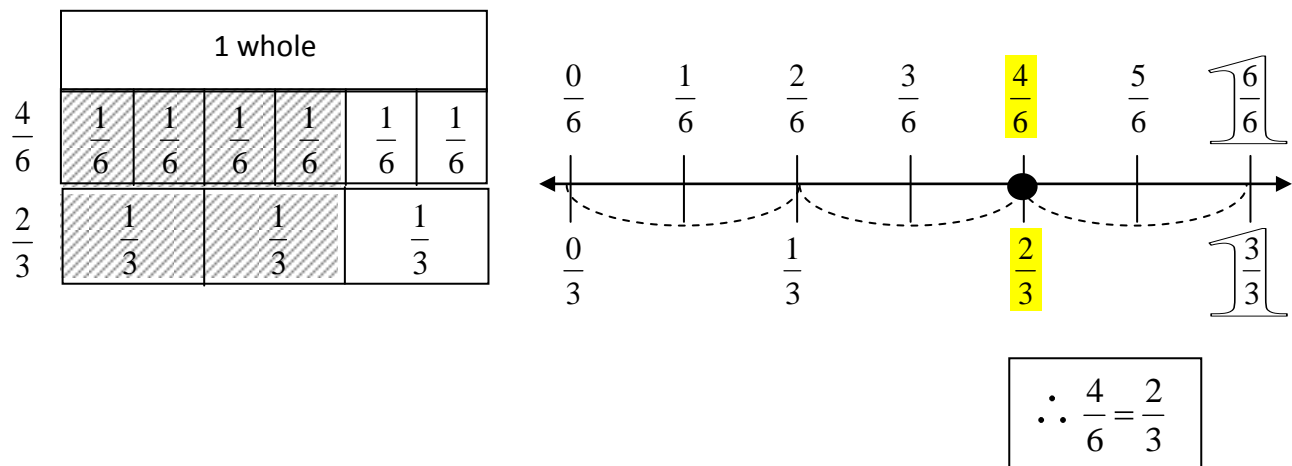


When we make our number line, we can start with sixths, or 6 equal parts. Then we can find 3 equal parts and label them thirds.



(or) (Establish sixths. Then run your finger across until you get 1 third across. Then mark that as 1 third. Do the same for 2 thirds.)

**We Do:** How many thirds are in 4 sixths?  $\frac{4}{6} = \frac{?}{3}$



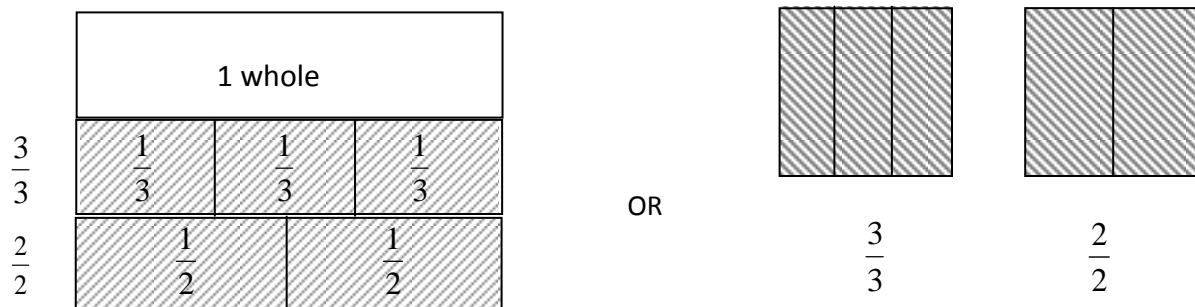
**You Try:** How many fourths are in 2 eighths?  $\frac{2}{8} = \frac{?}{4}$

**Other You Tries:** Include the fraction pairs used in previous lessons. You can have them do both bar model and number line or just number line.

**Stretch:** How many halves are in 3 thirds?  $\frac{3}{3} = \frac{?}{2}$  Draw a bar model.

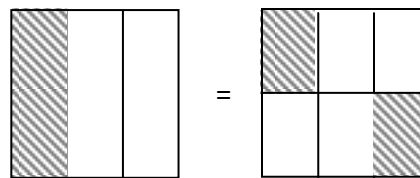
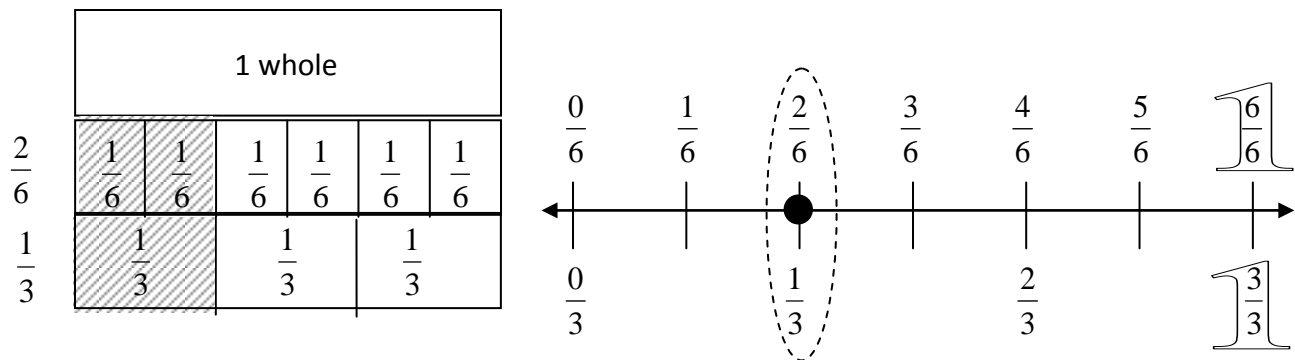
This example draws on their understanding of 1 whole and is more challenging to draw.

Students can draw the bar model or a different model. They can also explain.



$\frac{3}{3} = \frac{2}{2}$  because they are both equal to 1 whole.

## Connect Drawing, Bar Model, and Number Line

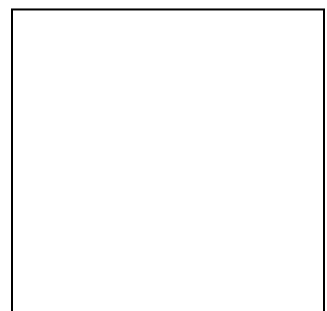
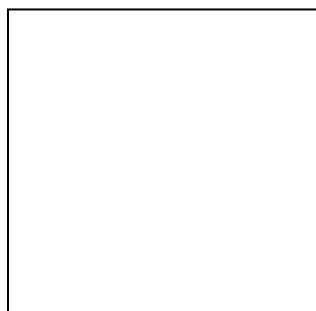
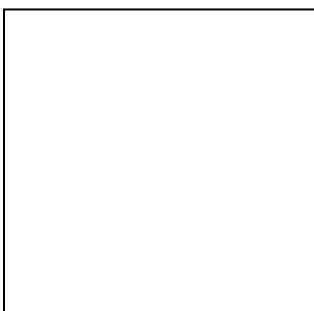
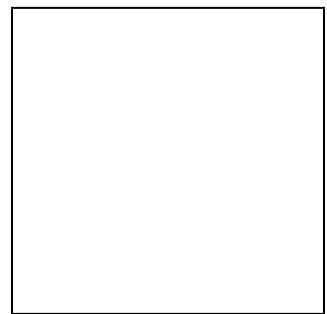
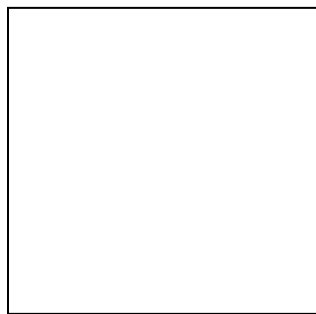
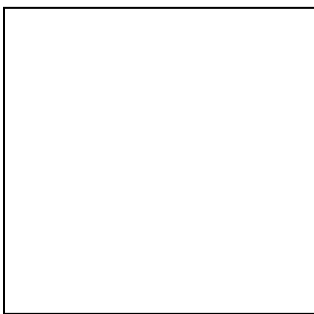
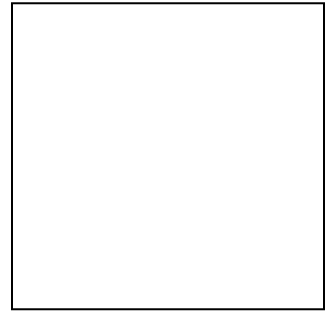
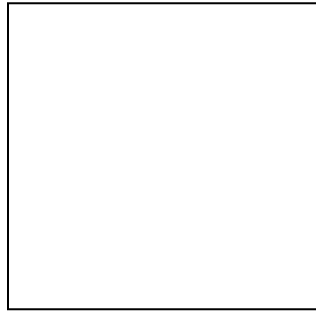
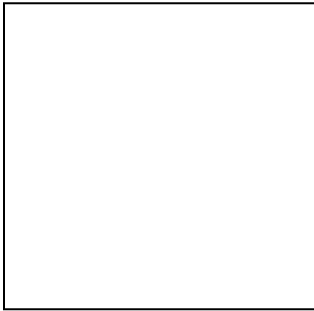
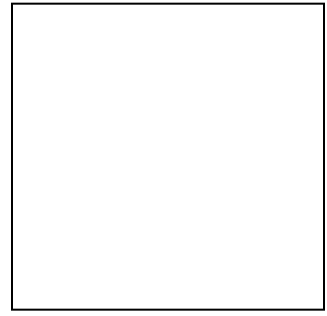
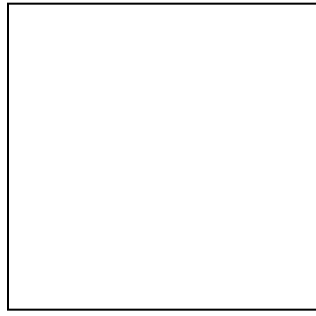
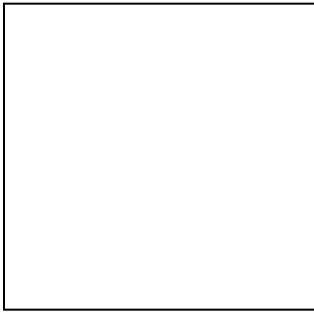


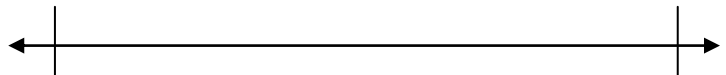
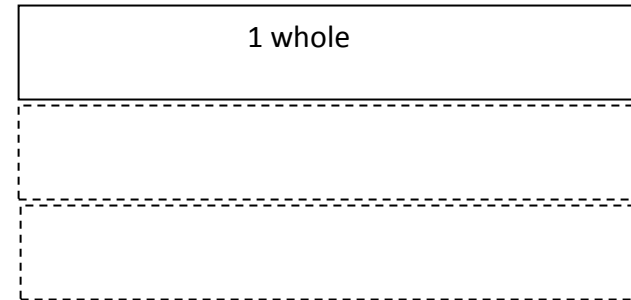
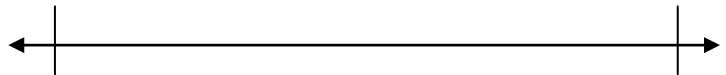
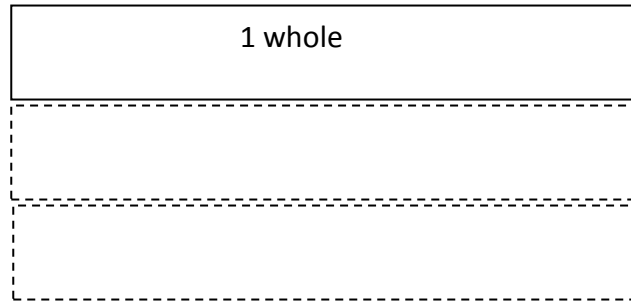
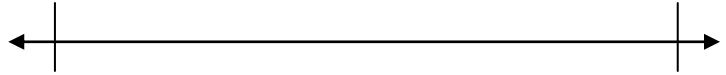
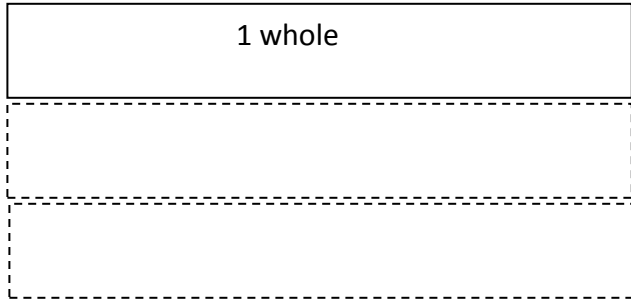
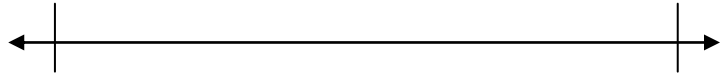
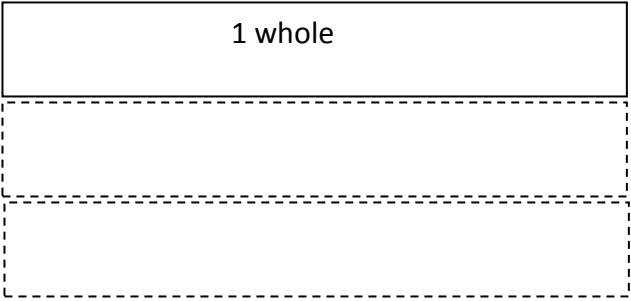
**Compare:** What is the same? What is different? Which models are most alike?

**Evaluate:** If you had to pick one model, which would you choose? Why?

Which is easiest to understand? Which is the most difficult?

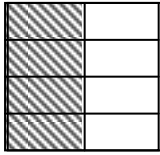
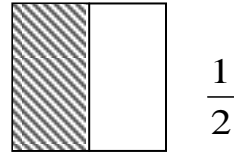






**Assesement:**

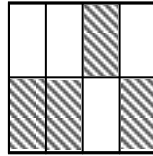
1) Circle all the models that are shaded  $\frac{1}{2}$ .



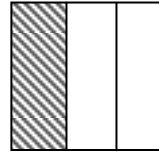
A



B

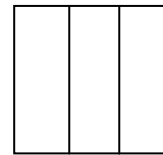
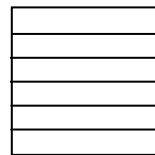
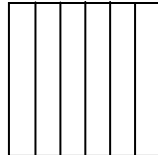
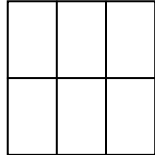
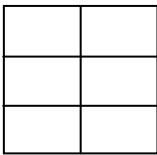


C



D

2) Shade in  $\frac{1}{3}$  of each model.

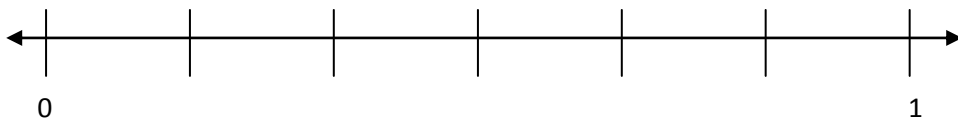


3) How many eighths are in 1 fourth?  $\frac{1}{4} = \frac{?}{8}$

Draw a model and a number line to prove your answer.

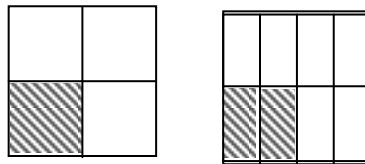
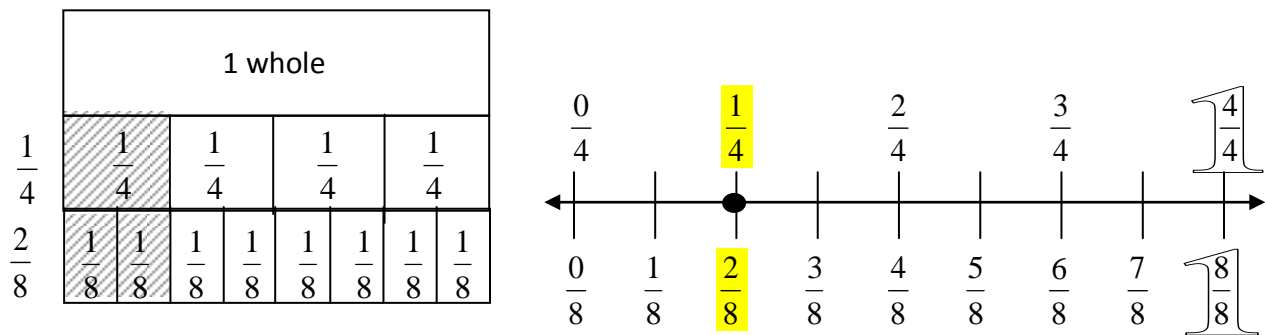
4) Label the number line below with the fractions in the box:

$\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{6}$ ,  $\frac{6}{6}$

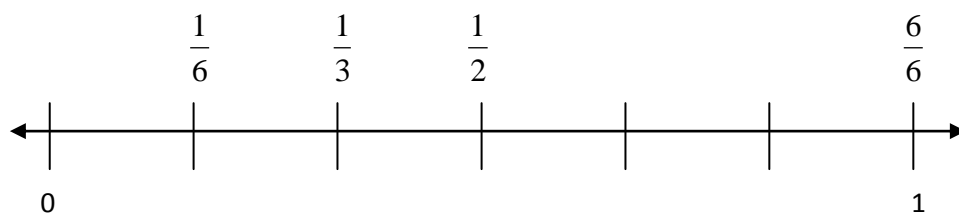


Assessment Rubric:

- 1) A, B, and C are all correct. Students that choose (D) do not understand the role of the denominator.
- 2) Students are able to generate equivalent fractions. Students that can successfully show  $2 \text{ eighths} = 1 \text{ fourth}$  with either a model or number line show proficiency.



- 3) Students show deep level of understanding, able to identify thirds, sixths, and halves on one number line.



## Warm Up

### Selected Response 3.OA.5

Which expression has the same value as  $8 \times 9$ ?

A)  $8 \times 3 \times 3$

Y

N

B)  $8 \times (3 + 3)$

Y

N

C)  $(8 \times 5) + (8 \times 4)$

Y

N

D)  $(8 \times 6) + (8 \times 3)$

Y

N

Done?

Complete this equation:  $8 \times 9 = 8 \times ( \underline{\quad} + \underline{\quad} )$

### Review 3.NBT.2

Find the difference using decomposition.  
Check your work with addition.

500	
395	?

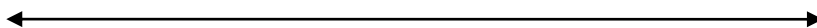
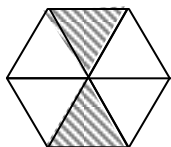
Challenge:

498	?
905	

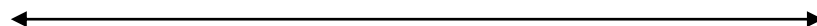
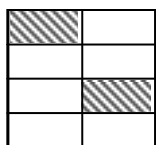
### Current 3.NF.2:

Create number lines to match the models.

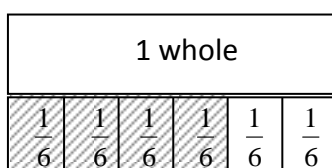
A)



B)



C)

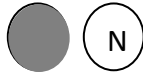


## Warm Up Debrief

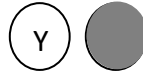
### Selected Response 3.OA.5

Which expression has the same value as  $8 \times 9$ ?

B)  $8 \times 3 \times 3$



C)  $8 \times (3 + 3)$



C)  $(8 \times 5) + (8 \times 4)$

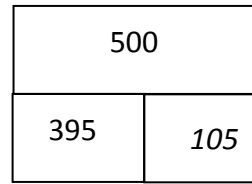


E)  $(8 \times 6) + (8 \times 3)$



### Review 3.NBT.2

Find the difference using decomposition.  
Check your work with addition.



*I'm looking for the difference, a missing part.*

$$\begin{array}{r} 500 \rightarrow 499+1 \\ -395 \rightarrow -395 \\ \hline 104+1 \\ = \textcircled{105} \end{array}$$

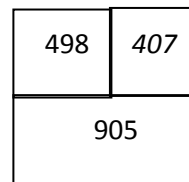
Check:

$$\begin{array}{r} 11 \\ 395 \\ +105 \\ \hline 500 \end{array}$$

Check:

$$\begin{array}{r} 407 \\ +498 \\ \hline 905 \end{array}$$

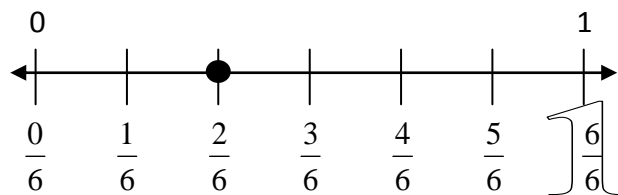
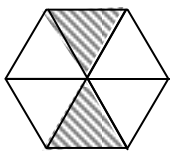
Challenge:



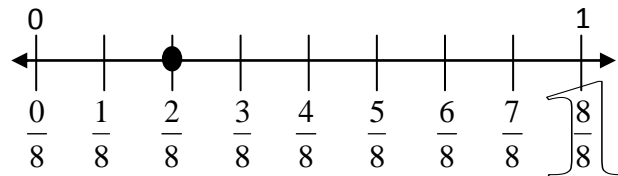
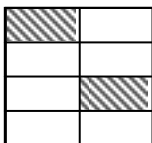
$$\begin{array}{r} 905 \rightarrow 900+5 \rightarrow 899+1+5 \\ -498 \rightarrow -498 \rightarrow -498 \\ \hline 401+1+5 \\ = \textcircled{407} \end{array}$$

### Current 3.NF.2 Create number lines to match the models.

A)



B)



C)

